Average Daily Balance and Compound Interest

Finite Math

21 February 2019

Quiz

If P dollars is invested in a savings account with an annual simple interest rate r, how much is in the account after t years?

$$A = ???$$

Average Daily Balance

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Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30*-day billing cycle is* $\$523.18$ *and purchases of* $\$147.98$ *and* $\$36.27$ *are posted on days* 12 *and* 25*, respectively, and a payment of* $\$200$ *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

Now You Try It!

Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

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Solution

\$708.92

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Example

Suppose $\$5,000$ *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

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- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $n =$ *total number of compounding periods*

Compound Interest

Alternately, one can reinterpret this formula as a function of time as

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A, P, r, and m have the same meanings as above and t is the time in years.

Compound Interest

Example

If $\$1,000$ is invested at 6% interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after 8 years? Round answers to the nearest cent.

Continuous Compound Interest

Consider again the formulation of compound interest given by

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&= P\left(1 + \frac{1}{x}\right)^{xrt} \qquad \left(x = \frac{m}{r}\right)
\end{aligned}$$

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Principal P invested at an annual nominal rate r will have future value

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Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.

Continuous Compound Interest

Example

If $\$1,000$ *is invested at* 6% *interest compounded continuously, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Now You Try It!

Example

If $\$2,000$ is invested at 7% compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) continuously, what is the amount after 5 years? Round answers to the nearest cent. (Assume 365 days in a year.)

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If $\$2,000$ is invested at 7% compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) continuously, what is the amount after 5 years? Round answers to the nearest cent. (Assume 365 days in a year.)

Solution

(a) \$2805.10
(b) \$2829.56
(c) \$2835.25
(d) \$2838.04
(e) \$2838.14